Study on natural frequencies of a real and small-scale model of a cylindrical tank

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1. Introduction

Comprehensive and in-depth understanding of complex dynamic induced behavior of structures usually requires the performance of numerically and laboratory experiments. This is especially important in the case when the dynamic behavior of the structure is strongly influenced and determined by the presence of water. Usually, the models are scaled versions of a real full-size structure. Scaling the real structure to a size desired and convenient for a laboratory test follows some well-known laws about geometric, kinematic and dynamic similitude.

This work presents an attempt to study numerically the relation between the dynamic characteristics of a real structure and a small-scale model, as well as to identify the influence of water density on the natural frequencies. The aim is to understand if the relation between the natural frequencies of the real structure and its scaled model remains the same as in the proposed similitude law and how it is influenced by the water density.

2. Physical modeling

A thin-walled steel vertical-axis cylindrical tank, anchored at the base is adopted, Fig. 1. It has a simple geometry but exhibits complex vibration behavior. Geometry: height of the tank (H=15.79m); water depth (h=14.63m); radius (R=27.42m); mean wall thickness (32.16mm). Material properties of the steel: Young modulus ($2.1*10^{11}$N/m^2); Poisson ratio (0.3); Density (7850kg/m^3). Material properties of water: Density (1000kg/m^3); Sonic velocity (1447m/s) at 10^{0}C.

Fig. 1. Water tank

3. Theoretical prediction and finite element analysis

The accepted approach consists in application of the classical geometric scaling law. This law is usually derived in textbooks by employing the method of dimensional analysis [1] and requires that all scaled dimensions are proportional. Strictly speaking, it also is required that the Poisson's ratios be the same if two different materials are used: $v_2=v_1$, where the subscripts 1 and 2 designate both similar structures. In experimental model work, this condition is often relaxed if Poisson's ratios are approximately equal or equal when structure 1 and 2 are built from the same material.

The natural frequencies of structure 2 are then related to the natural frequencies of structure 1, as proposed in [2], by:

$$\omega_{k2} = \omega_{k1}\frac{a_1}{a_2}\sqrt{\frac{\rho_1 E_2}{\rho_2 E_1}} = \omega_{k1}\frac{a_1}{a_2}\frac{c_2}{c_1} \quad \textbf{(1)}$$

where ρ is the mass density, E is Young's modulus, c is the speed of sound, and α is a typical length or width dimension of the structure.

To obtain the natural frequencies numerically a finite element simulation was performed within ANSYS(R) software environment, [3]. Following the general principles of the FEM for discretization of complicated structures, the numerical model was generated, Fig. 2. Crucial for the numerical analysis is the selection of suitable elements, since it directly influences the computational time and accuracy of the results.

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Key words: Modal analysis, Tank, Water-Structure Interaction

4. Results and discussion

To validate the model approach established in ANSYS environment, a comparison was made between the fundamental frequency extracted numerically for the case of an empty and water-filled tank, incl. sloshing and impulsive mode, and the one calculated by a particular formula [2,4], as well as with that given in [5,6]. The results showed a good match.

Further, the natural frequencies of the adopted real tank were numerically extracted. The same analysis was performed for the 10 times smaller scaled model (the scale factor was arbitrarily chosen). According to Eq. (1) the natural frequencies of the small scaled model are expected to be 10 times bigger than those of the real tank under the condition of scaling all structural parameters. The expectation was confirmed by the numerical analysis, and the results regarding the impulsive mode of the water-filled tank obtained by ANSYS are shown in the table.

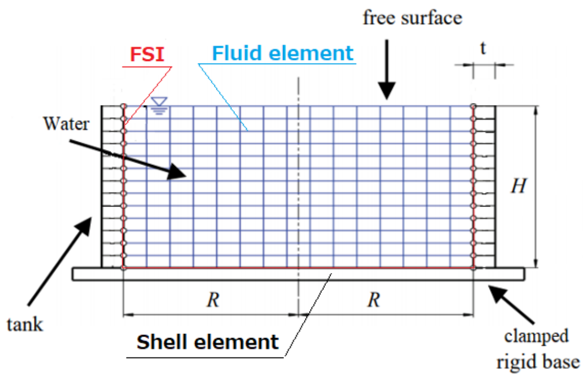

Fig. 2 FE model

Mode	Real geom.	Scaled 1/10
1	1.0155	10.151
3	1.0194	10.192
5	1.0526	10.523

5. Conclusion

The frequencies, extracted by numerical simulation, of a real scale and small scaled model of the adopted water-filled cylindrical steel tank confirmed the expected theoretically determined result and relation, i.e. the scale factor in Eq. 1. So, utilizing the small model, that represents the scaled geometrically analog of the real structure, allows to obtain the dynamic characteristics of the real structure by dynamic experiments. The results showed that strictly following of the geometry scaling law, Eq. 1, has not demanded any changes of the fluid density between the real and small-scale models. The scaling law, Eq. 1, however, proved to be sometimes too restrictive since it was not always convenient, for instance, to scale the wall thickness in proportion to typical surface dimensions. That is essential especially in the case of laboratory tested thin-walled fluid-structure models, where the rigorous application of the scale law may lead to structurally impossible geometry that would endure local deformation and buckling even before performing the laboratory dynamic test.

In that case, varying the density of the fluid would allow to keep the wall thickness within a structurally reasonable dimension and thus to model the actual dynamic characteristic of the structure. Task for future research is to perform laboratory tests to establish and verify the relation between the fluid density, scaled model geometry and the dynamic characteristics of the real full-scale structure in case when the geometry scaling law could not be strictly applied, i.e. the wall thickness of the real structure is thin and its scaling in accordance of the law is not possible. The established relation would be useful for the proper design and carrying out of dynamic tests in laboratory conditions, especially regarding the dynamic-induced behavior of structures where neither the structure nor the fluid alone govern the response, and which are highly sensitive to frequency characteristics of the dynamic excitation.

6. Reference

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Acknowledgement: This study was supported by JSPS Postdoctoral Fellowship - FY2017