クーロン土圧の再考
REVIEW OF COULOMB'S EARTH PRESSURE THEORY

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Introduction: Earth pressure is the lateral force exerted by the soil on retaining system. It is dependent on interaction or movement with the retaining structure. The approaches to the design of retaining structures need to ensure that total collapse or failure does not occur. There are two commonly accepted methods for calculation of simple earth pressure such as Coulomb's and Rankine's theory. The Coulomb theory was developed in the 1776 and the Rankine theory was developed in 1857 and remains as the basic for earth pressure calculation in the present day. In practice, it can be said that the Coulomb's theory is more advantageous than Rankine's theory. The present study proposed to contribute another form of general equation for active and passive earth pressure on retaining structure in terms of soil strength parameter based on Coulomb's theory.

Coulomb's theory: The well known earth pressure theories of Rankine and Coulomb provide expression for the active and passive pressure for a soil mass at a state of failure. When a state of soil failure has been reached, active and passive failure zones became approximated by straight planes. Using identical parameters, the Coulomb wedge theory calculates less earth pressure than the Rankine's theory for a level of back slope whereas the value converges under back slope conditions. The Coulomb's theory calculates a unique failure angle for every design condition where as application of Rankine theory to reinforced soil structures fixes the internal failure plane at 45+ϕ/2.

The Coulomb's theory provides a method of analysis that gives the resultant horizontal force on a retaining system for any slope of wall, friction, and slope of backfill provided $\beta \leq \phi$. This theory is based on the assumption that soil shear resistance develops along the wall and failure plane. The active and passive earth pressures are determined using the following equation with resultant pressure acting at angle δ. For a level ground condition, Coulomb's equation will be the same as Rankine's equation.

$$K_a = \frac{\sin^2(\phi - \omega)}{\left(\sin^2 \omega\right)\{\sin(\delta + \omega)\}\left[1 + \sqrt{\frac{\{Sin(\phi + \delta)\}\{Sin(\phi - \beta)\}}{\{\sin(\delta + \omega)\}\{\sin(\beta - \omega)\}}}\right]^2} \quad \text{..........(1)}$$

Present study: A general way of the forces acting on a retaining system is depicted in Fig.1. As can be seen from this figure, a wedge of soil ABC tends to slide over a plane of failure, which tilts forward the wall. At this time, it considered that the horizontal and vertical component of internal force H and V are acting at the vicinity of the wedge ACD and DCB in opposite direction. Also the shear force (T) and the normal force (N) are taken at the failure plane, l is defined as the length of failure plane, W_0 and W_1 are weight of the backfill soil of ΔACD and ΔDCB respectively. The angle of friction between the soil and the wall is represented by δ. The value of δ is assumed as1/3 to 2/3 of ϕ, β is the inclination angle of backfill soil with respect to horizontal component and the angle of the wall (ω) is measured from horizontal component. Total weight of the retaining structure is denoted by W and expressed as,

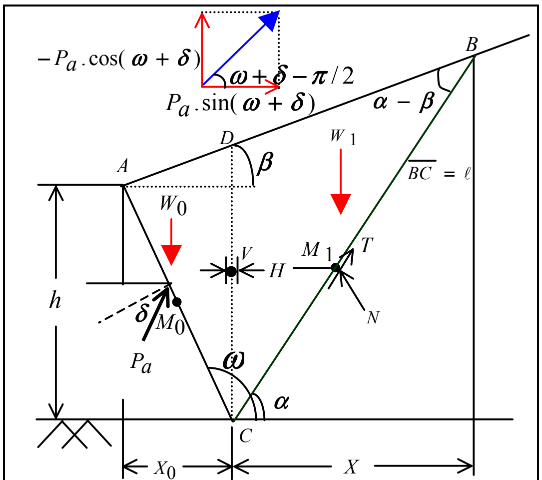

Fig.1. Force diagram in Retaining System

$$W = W_0 + W_1 = \frac{1}{2}\gamma h^2 \frac{Sin(\omega - \beta).Sin(\omega - \alpha)}{Sin^2(\omega).Sin(\alpha - \beta)} \quad \text{.......(2)}$$

$$l = \overline{BC} = h\frac{Sin(\omega - \beta)}{Sin(\omega).Sin(\alpha - \beta)} \quad \text{.......(3)}$$

Length l is mentioned in equation (3). The force equilibrium of wedge ΔABC is considered as shown in Fig.1. In ΔACD, vertical and horizontal component of internal force can be written as,

$$V = -W_0 - P_a.Cos(\omega + \delta) \quad \text{.......(4)} \qquad H = P_a.Sin(\omega + \delta) \quad \text{.......(5)}$$

Also for ΔDCB, the force equilibrium equation can be expressed as,

$$V - W_1 + N.Cos(\alpha) + \tau.Sin(\alpha) = 0 \text{ ....(6)} \quad H - N.Sin(\alpha) + \tau.Cos(\alpha) \text{......(7)} \quad T = c'/l + N.\tan(\phi') \text{ ....(8)}$$

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Determining N from equations (6), (7) and (8) and substituting the value of N to equations (4) and (5), equation (9) is obtained. This equation is general equation for active and passive earth pressure, which is defined in terms of strength parameters c' and ϕ. Applying the condition $dP_a / d\alpha = 0$ to equation (9), the following equation is obtained.

$$P_a = \frac{W.Sin(\alpha - \phi) - c'.l.Cos(\phi)}{Sin(\omega + \delta + \phi - \alpha)} \quad ....(9)$$

$$Sin(\omega + \delta + \phi' - \alpha)\left\{\frac{2c'}{\gamma h}.Sin(\omega).Cos(\phi').Cos(\alpha - \beta) - Sin(\omega - \beta).Sin(\alpha - \phi')\right\}$$

$$- Sin(\alpha - \beta)\left\{\frac{2c'}{\gamma h}.Sin(\omega).Cos(\phi').Cos(\omega + \delta + \phi' - \alpha) - Sin(\omega + \delta).Sin(\omega - \alpha)\right\} = 0 \quad ....(10)$$

According to equation (10), λ and $tan\alpha$ can be defined as equation (11) and (12).

$$\lambda = \sqrt{\frac{\frac{2c'}{\gamma h}.Sin(\omega).Cos(\phi') + Sin(\omega - \beta).Sin(\phi' - \beta)}{\frac{2c'}{\gamma h}.Sin(\omega).Cos(\phi') + Sin(\omega + \delta).Sin(\delta + \phi')}} \quad ....(11) \qquad \tan\alpha = \frac{s(\beta) + \lambda.s(\xi)}{c(\beta) + \lambda.c(\xi)} \quad ....(12)$$

At the same time, equations (10), (11) and (12) are applied in equation (9) and hence, other general form of earth pressure equations can be obtained as follow,

$$P_a = \frac{1}{2}\gamma h^2 \frac{Sin^2(\omega - \phi')}{Sin^2(\omega).Sin(\omega + \delta)}\left[1 + \sqrt{\frac{Sin(\phi' - \beta).Sin(\delta + \phi')}{Sin(\omega - \beta).Sin(\omega + \delta)}}\right]^{-2} \quad ....(13)$$

$$P_a = \frac{1}{s(\omega + \delta)}.\frac{1}{2}\gamma h^2 \frac{s^2(\omega - \beta)}{s^2(\omega)}\left\{\frac{s(\omega + \delta).\lambda - s(\phi' - \beta)}{s(\omega + \delta + \phi' - \beta).\lambda}\right\}^2 \quad ....(14)$$

In the final analysis, the general equation of the active earth pressure can be obtained in the different form, such as equation (13) and (14). On the other hand, the method presented here is to propose the analytical method for determination of earth pressure.

Result and Discussion: To verify the applicability of the present method, the active and passive earth pressure and inclination angle of failure plane with respect to horizontal component are determined by using equations (9), (11) and (12). The soil strength parameters are shown in Table 1. Fig.2 depicts the inclination angle of failure plane and analytical results are shown in Table 2. As it is clear from this table, when the cohesion of soil c = 19.62 KN/m^2, the active earth pressure is approach to zero and this situation is said to be self standing point of retaining system. From the analytical result, if the cohesion of soil is increased, the passive earth pressure also gradually increased. While the cohesion of soil is zero, active earth pressure is not equal to zero but very small and passive earth pressure is minimum in this condition. It can be pointed out a weak point of the Coulomb's theory in which the cohesion of soil is assumed to be zero in practical design. But, this condition is not appropriate in actual condition. It is guessed that in real design condition, it is better to consider the effect of cohesion of soil for approaching safety design. In the case of c >>0 with comparatively larger inclination angle, it is observed that the passive earth pressure increases gradually with the increase in cohesion of soil.

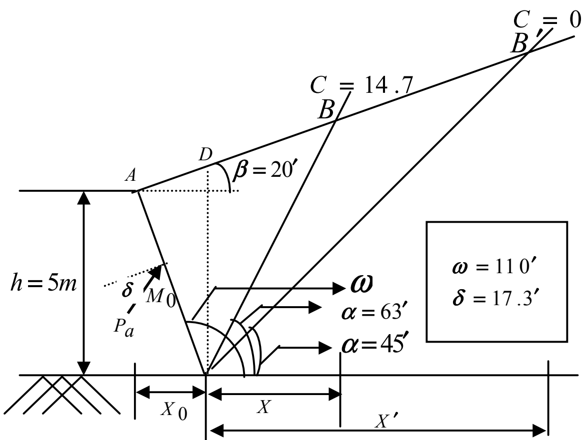

Fig.2. Scaled diagram of Earth pressure

$\omega = 110'$ $\quad h = 5m$

$\beta = 20'$ $\quad \gamma = 1.8 * 9.81\ KN/m^2$

$\phi = 26'$ $\quad c = 0\ KN/m^3$

$\delta = \frac{2}{3}\phi$ $\quad c = 1.5 * 9.81\ KN/m^3$

Table .1. Analytical data

$c=0$				$c=14.7$				$c=19.6$			
Pa	α	*Pp*	α	*Pa*	α	*Pp*	α	*Pa*	α	*Pp*	α
18.7	44	139.5	43	5.9	63	185	43	2.5	66	200.5	43

Table .2. Analytical result data

References 1. Soil mechanics, Tien Hsing Wu, Bostan 1966, Chapter 9. Earth-Pressure Problems., 2. Series in Soil Engineering, Soil Mechanics, SI Version T.William Lambe. Robert V.Whitman, Chapter 13,Earth-retaining Structures and Slopes, 3.Trenching and shoring Manual Issued by Division of Structure Construction, 1990