レーダトモグラフィにおける誘電率・導電率推定のための全波形逆解析手法
Full-waveform inversion of radar tomography data to estimate distribution of electrical permittivity and conductivity

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1. Introduction

Geophysical prospecting method using high frequency electromagnetic wave like ground penetrating radar or radar tomography is efficient for geological survey, hydrology and environmental study, and non-destructive test for structure.

Wang et al. (1994) presented an imaging algorithm to produce interwell conductivity distribution from time-domain or transient electromagnetic (TEM) data. Their imaging was based on a method of full waveform inversion, which is first developed for seismic wavefields by Tarantola (1984). Though Sanada et al. (1999a) applied the Wang et al's (1994) algorithm to EM tomography for electrical conductivity estimation high frequency, the methods of full-waveform inversion for permittivity estimation has not been discussed.

This paper presents an imaging algorithm for GPR data by applying the full-waveform inversion to EM fields. Especially we emphasize the inversion process for permittivity estimation which has not been considered.

2.Theory and Inversion Procedure.

Consider a GPR survey in which an EM source such as an electric dipole is energized with a source waveform $S(t)$, and the transient electric field is measured at positions $\mathbf{r}_i$ (i = 1, 2,…, NR) from time $t = 0$ to $t = T$. Let $\mathbf{e}_0(\mathbf{r}_i, t)$ denote these measurements and assume all fields are zero for $t < 0$. The goal of GPR inversion is to find a model [a conductivity distribution $s(\mathbf{r}')$ and/or a permittivity distribution $e(\mathbf{r}')$] such that the transient electric field $\mathbf{e}(\mathbf{r}_i, t)$ calculated for the model fits the observed field in some sense. From here on, let $\mathbf{r}'$ and $\mathbf{r}$ denote points in the region where the model is allowed to vary and in the region where measurements are made, respectively. The simple choice is to match the data in a least-squares sense, which corresponds to finding a model that minimizes the error functional

$$R_d(\mathbf{p}) = \frac{1}{2}\sum_{i=1}^{NR}\int_0^T [\mathbf{e}_0(\mathbf{r}_i,t) - \mathbf{e}(\mathbf{r}_i,t)]^2 dt = \frac{1}{2}\sum_{i=1}^{NR}\int_0^T d\mathbf{e}_0(\mathbf{r}_i,t)\cdot d\mathbf{e}_0(\mathbf{r}_i,t)dt \qquad (1)$$

steepest descent algorithm. Let g denote the gradient of R_d with respect to a model $\mathbf{p}$, i.e.,

$$g = dR_d / d\mathbf{p}, \ (g_s = dR_d / ds, \quad \text{and/or} \quad g_e = dR_d / de). \qquad (2)$$

When model parameters are changed to $\mathbf{p}' = \mathbf{p} + d\mathbf{p}$, EM fields vary to $\mathbf{e}' = \mathbf{e} + d\mathbf{e}$ and $\mathbf{h}' = \mathbf{h} + d\mathbf{h}$

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Dropping terms involving the product of two perturbed quantities gives

$$\left(s + e\frac{\partial}{\partial t} \right) d\mathbf{e} - \nabla \times d\mathbf{h} = -\left(ds + de\frac{\partial}{\partial t} \right)\mathbf{e}, \quad \nabla \times d\mathbf{e} + m\frac{\partial d\mathbf{h}}{\partial t} = 0, \tag{3}$$

The solution of these equations is given by

$$d\mathbf{e}(\mathbf{r}_i, t) = \int_{V'}\int_0^t \mathbf{g}_{11}(\mathbf{r}_i, t \mid \mathbf{r}', t') \cdot \left[ds(\mathbf{r}') + de(\mathbf{r}')\frac{\partial}{\partial t'} \right] \mathbf{e}(\mathbf{r}', t')\, d\mathbf{r}'\, dt' \tag{4}$$

, where $\mathbf{g}_{ii}(\mathbf{r}, t \mid \mathbf{r}', t')$ is a dyadic Green function

From the reciprocity relation, we have

$$g_s(\mathbf{r}') = \int_0^T dt'\, \mathbf{e}(\mathbf{r}', t') \cdot \mathbf{e}_b(\mathbf{r}', t' \mid d\mathbf{e}_0),\; g_e(\mathbf{r}') = \int_0^T dt' \frac{\partial}{\partial t'}\mathbf{e}(\mathbf{r}', t') \cdot \mathbf{e}_b(\mathbf{r}', t' \mid d\mathbf{e}_0) \quad \text{where} \quad \mathbf{e}_b(\mathbf{r}', t' \mid d\mathbf{e}_0) = \sum_{i=1}^{NR}\int_T^{t'} dt\, \mathbf{g}_{11}^{+}(\mathbf{r}', t' \mid \mathbf{r}_i, t) \cdot d\mathbf{e}_0(\mathbf{r}_i, t) \tag{5}$$

$\mathbf{e}_b$ is the back-propagated field starting from $t=T$. The adjoint Green dyadic $\mathbf{g}_{11}^{+}$ gives the electric field at $\mathbf{r}'$ and time t' caused by an electric current source $d\mathbf{e}_0$ radiating at $\mathbf{r}_i$ at a later time t.

The actual process of the inversion is

1) Compute its electric fields $\mathbf{e}(\mathbf{r}', t'; \mathbf{s}_j)$ through forward modeling using FDTD simulation .
2) Calculate $\mathbf{e}_b(\mathbf{r}', t' \mid d\mathbf{e}_0(\mathbf{s}_j))$ by back-propagating the electric field difference $d\mathbf{e}_0(\mathbf{r}_i, t; \mathbf{s}_j)$ with the adjoint dyadic $\mathbf{g}_{11}^{+}$.
3) Update the current model with $g(\mathbf{r}')$ obtained from the cross-correlation of $\mathbf{e}(\mathbf{r}', t'; \mathbf{s}_j)$ and $\mathbf{e}_b(\mathbf{r}', t' \mid d\mathbf{e}_0(\mathbf{s}_j))$ using conjugate gradient method
4) Repeat the process 2) – 4) until a convergence is attained.

3. Result

Fig.1 shows the result which simulated cross-borehole radar tomography survey applied to the ground with high permittivity anormally. The more efficient analysis method for this inversion and the comparison to conventional methods like ray-path analysis will be discussed.

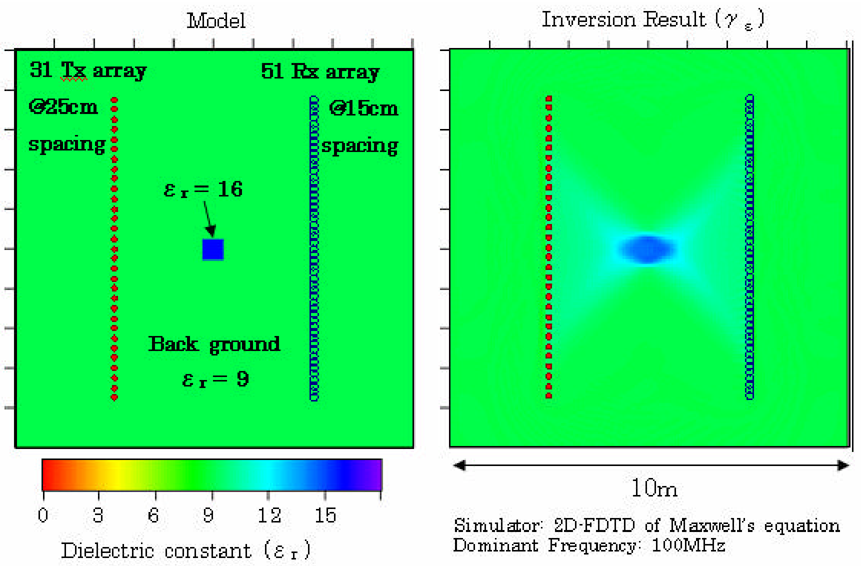

Fig.1: Full wave inversion results for cross hole radar tomography simulation

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