Application of Distributed Soil Moisture Model to Estimate Groundwater Recharge

–Case Study of Unconfined Shallow Groundwater in Imazu Area–

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1. Introduction

The concern about the groundwater is increasing day by day because it accounts for a major portion of the world's freshwater resources. Groundwater is contained in geological formations, called aquifers, which are sufficiently permeable to transmit and yield water. An unconfined aquifer is a layer of water-bearing material without a confining layer at the top of the groundwater, called the groundwater table. Groundwater recharge is the main source of water in unconfined aquifer. It is also the major transporting agent for contaminants through soil and the unsaturated zone. Thus, estimating groundwater recharge plays a vital role in the study related to groundwater management and pollutant contamination. Fazal *et al.*[1] have proved the potentiality of soil moisture accounting and routing model for estimating recharge using groundwater level data. But, the estimation is performed on lumped assumption, where physical characteristics of study area (e.g., landuse, land slope, surface geology etc.) are not considered. Distributed soil moisture model [2] is also capable to estimate recharge using those characteristics. Therefore, a study on horizontal movement of unconfined groundwater is explained, where the recharge is estimated by distributed soil moisture model.

2. Distributed soil moisture model

The distributed soil moisture model consists of two submodels, (i) a soil moisture submodel [3] and (ii) a tank submodel [4]. In this model structure, a domain of interest is divided into many square-size cells, and again cell profile is divided into three zones, e.g., surface zone, vadose zone and groundwater zone. In soil moisture model, the rainfall, evapotranspiration and infiltration of each cell interact with one another to produce surface runoff r_s and groundwater component r_g, and tank model transforms them from each zone of a cell into discharge. The schematic representation of the model in a plain area cell is shown in **Fig.1**. The groundwater component r_g can be treated as direct recharge, which is generated with r_s following below equations,

$$r_s = r_1 + r_2 + (1 - G)r_3 \tag{1}$$

$$r_g = Gr_3 \tag{2}$$

with

$$r_1 = H\frac{w_{act}}{w_{cap}}(h_s + R - E_p) \tag{3}$$

$$r_2 = \max\left\{(1 - H\frac{w_{act}}{w_{cap}})(h_s + R - E_p) - Y, 0\right\} \tag{4}$$

$$r_3 = \max\{I - (Z - W_{act}), 0\} \tag{5}$$

$$Y = Y_c + (Y_m - Y_c)e^{-\gamma\frac{w_{act}}{w_{cap}-w_{act}}} \tag{6}$$

$$I = \min\left\{(1 - H\frac{w_{act}}{w_{cap}})(h_s + R - E_p), Y\right\} \tag{7}$$

where r_1 is direct runoff, r_2 is surplus runoff beyond the infiltration, r_3 is infiltrated runoff component which is excess to the field capacity, G is groundwater runoff coefficient, H is model parameter related to direct runoff, h_s is surface storage, R is rainfall, E_p is potential evapotranspiration rate, Y is actual infiltration rate depending on soil moisture, I is actual infiltrated moisture, w_{act} is actual soil moisture depth within top five layers, w_{cap} is capacity depth of them, W_{act} is actual soil moisture content of all the layers, Z is soil moisture capacity depth at field capacity, Y_m is maximum infiltration rate and Y_c is steady infiltration rate.

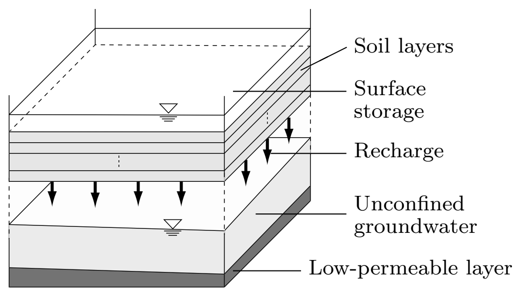

Fig.1 Schematic representation of a cell

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3. Unconfined groundwater flow

3.1 Governing equation The governing equation for horizontal flow in an isotropic, unconfined groundwater based on Dupuit's uniform flow approximation [5] is described as follows:

$$n^E \frac{\partial h}{\partial t} = -\nabla\cdot(-T\nabla h) + Q \quad (8)$$

with

$$T = (h - Z_r)K, \quad Q = r_g(t-t_p)-l \quad (9)$$

where n^E is effective porosity ($=\theta_s-\theta_r$), θ_s is saturated soil moisture content, θ_r is residual soil moisture content, h is water table level, T is transmissivity, Z_r is low-permeable layer level, K is saturated hydraulic conductivity, Q is net source of water, t_p is time lag factor and l is leakage.

Initial and boundary conditions are added to solve the above partial equation. The initial condition is defined as

$$h(\boldsymbol{x}, 0) = h_0(\boldsymbol{x}) \quad \text{in} \quad \Omega \quad (10)$$

where $h_0(\boldsymbol{x})$ is prescribed head of $\boldsymbol{x}$. The boundary conditions are given by

$$h(\boldsymbol{x}, t) = h^*(\boldsymbol{x}, t) \quad \text{on} \quad \Gamma_D \quad (11)$$

$$T\frac{\partial h}{\partial \boldsymbol{\nu}} = q(\boldsymbol{x}, t) \quad \text{on} \quad \Gamma_N \quad (12)$$

where $h^*(\boldsymbol{x}, t)$ and $q(\boldsymbol{x}, t)$ are prescribed water table level and flux of $\boldsymbol{x}$ and t, respectively, $\boldsymbol{\nu}$ is unit outward normal vector of Ω, Γ is boundary of the domain Ω, D and N indicate the Dirichlet and Neumann boundaries, respectively.

3.2 Numerical model To obtain a FVM (Finit Volume Method) model of eq. (8), both sides are multiplied by weighting function W_e and integrated in the whole domain Ω as following equation:

$$\int_\Omega W_e n^E \frac{\partial h}{\partial t} d\Omega = \int_\Omega W_e \nabla\cdot(T\nabla h) d\Omega + \int_\Omega W_e Q d\Omega \quad (13)$$

with

$$W_e = \begin{cases} 1 & \boldsymbol{x} \in \Omega_e \\ 0 & \boldsymbol{x} \notin \Omega_e \end{cases} \quad (14)$$

where n^E, h and Q are assumed to be constant in Ω_e, i.e., those are described as n_e^E, h_e and Q_e, respectively. Divergence theorem of Gauss is applied, and the eq. (13) is formulated as follows.

$$\Delta_e n_e^E \frac{dh_e}{dt} = f_e + \Delta_e Q_e \quad (15)$$

with

$$\Delta_e = \int_{\Omega_e} d\Omega, \quad f_e = \int_{\Gamma_e \setminus \Gamma_N} T\nabla h \cdot \boldsymbol{\nu_e} d\Gamma_e, \quad \Gamma_e = \partial\Omega_e \quad (16)$$

Therefore, the ordinal differential equation is obtained which is solved by implicit scheme such as Picard method after assembling all cell-equations.

$$\frac{d\boldsymbol{h}}{dt} = \boldsymbol{b} \quad (17)$$

with

$$b_e = \frac{1}{n_e^E}\left(\frac{f_e}{\Delta_e} + Q_e\right) \quad (18)$$

4. Application

A part of Imazu-cho (12.75 km^2), located between the Ishida river and the Sakai river of Shiga prefecture, is selected as a study area for this study. The major landuse types of the site are forest, paddy field and city (**Fig.2**). Most of the area is covered with flat land which is used for human and commercial activity. Thus, the surface and subsurface flows of this area have the potentiality of pollutant contamination. Our interest is expressed about the subsurface unconfined shallow groundwater flow. The study area is digitized from a satellite image and divided into square-size cells with a size of 100m × 100m. Clay-level, which is low-permeable layer of the study area is obtained by interpolation of ten-boring data. The simulated unconfined shallow groundwater flow of this area will be shown in the presentation.

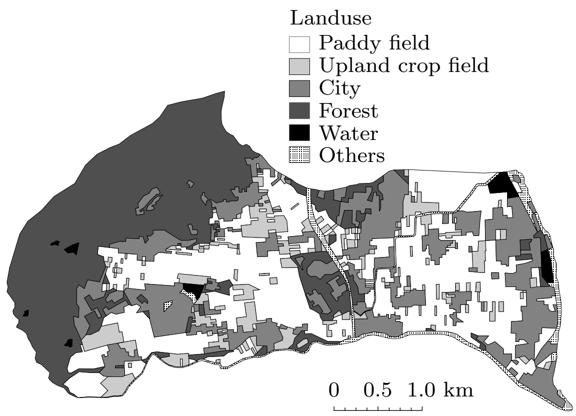

Fig.2 Landuse of study area

5. Conclusion

Numerical model of unconfined shallow groundwater flow is described to determine the water flow and depth of water table in the study area, where water quality management is supposed to be considered.

References [1] Fazal, M.A. *et al.* (2005), *J.Hydrol.*, 303, pp.56-78. [2] Alam, A.H.M.B. *et al.* (2005), *JIDRE.* (Submitted) [3] Tan, B.Q. and O'Connor, K.M. (1996), *J.Hydrol.*, 185, pp.275-295. [4] Suzuki, T. *et al.* (1996), *Civil Engineering J.*, 38(10), pp.26-31. [5] Huyakorn, P.S. and Pinder, G.F. (1983), Computational methods in subsurface flow. Academic Press, Inc. New York.